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chords and the intercepted arcs. $\therefore$ arc $HP=\frac{1}{2}$ arc AP, and arc $PK=\frac{1}{2}$ arc PC. $\therefore$ arc $HPK=\frac{1}{2}$ arc APC. But arc APC measures $\angle F=60^\circ$ at circumference; therefore HK, its half, measures an equal angle at the center, and $\angle O_2=\angle F=60^\circ$. Similarly, $\angle O_1$ may be shown$=\angle E$, and $\angle O=\angle D$. Since equiangular triangles are equilateral, OO_1O_2 is equilateral. Q. E. D.

Solved in a similar manner by *G. I. HOPKINS*, *F. R. HONEY*, *J. SCHEFFER*, *C. A. JONES*, and *G. B. M. ZERR*.

CALCULUS.

73. Proposed by MOSES COBB STEVENS, A. M., Professor of Mathematics, Purdue University, Lafayette, Ind.

Solve $\int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

$$\int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx=\int_0^{\frac{1}{4}\pi} \log[1+\tan(\tfrac{1}{4}\pi-x)]dx=\int_0^{\frac{1}{4}\pi} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$=\int_0^{\frac{1}{4}\pi} \log\left(\frac{2}{1+\tan x}\right)dx=\int_0^{\frac{1}{4}\pi}\log 2.dx-\int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx.$$

$$\therefore\ 2\int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx=\int_0^{\frac{1}{4}\pi} \log 2.dx=\tfrac{1}{4}\pi\log 2.$$

$$\therefore \int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx=\tfrac{1}{8}\pi\log 2.$$

(See *Todhunter's Integral Calculus*, Art. 51, page 66.)

II. Solution by T. A. CLARK, of the Senior Class, Purdue University, Lafayette, Ind.

$$\int_0^{\frac{1}{4}\pi} \log(1+\tan x)dx=\int_0^{\frac{1}{4}\pi} \log(\tan\tfrac{1}{4}\pi+\tan x)dx=\int_0^{\frac{1}{4}\pi} \log\frac{\sin(\frac{1}{4}\pi+x)}{\cos\frac{1}{4}\pi\cos x}$$

$$=\int_0^{\frac{1}{4}\pi} \log\sec\tfrac{1}{4}\pi dx+\int_0^{\frac{1}{4}\pi} \sin(\tfrac{1}{4}\pi+x)dx-\int_0^{\frac{1}{4}\pi} \log\cos x dx=\tfrac{1}{4}\pi\log\sqrt{2}.$$

To prove $\int_0^{\frac{1}{4}\pi} \log\sin(\frac{1}{4}\pi+x)dx-\int_0^{\frac{1}{4}\pi} \log\cos x dx=0$. Put $\frac{1}{4}\pi+x=\theta$, and

$\frac{1}{2}\pi-x=\phi$, then $\int_0^{\frac{1}{4}\pi} \log\sin(\frac{1}{4}\pi+x)dx=\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \log\sin\theta d\theta$, and $\int_0^{\frac{1}{4}\pi} \log\cos x dx$

$$=-\int_{\frac{1}{2}\pi}^{\frac{1}{4}\pi}\sin\phi d\phi=\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}\sin\phi d\phi.$$ Therefore it is evident $\int_0^{\frac{1}{4}\pi}\log\sin(\frac{1}{4}\pi+x)dx$

$-\int_0^{\frac{1}{4}\pi}\cos x dx=0.$

74. Proposed by EDWARD R. ROBBINS, A. B., Mathematical Master in the Lawrenceville School, Lawrenceville, N. J.

A circular ring, whose radii are a and b, is cut by a plane making the area of the section (or sections) a maximum. Required the position of the plane, and the nature and area of the section (or sections).

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School. Chester, Pa.

Let the axis of the surface be taken as the axis of z, then the equation to the ring is

$$(x^2+y^2+z^2+ab)^2=(a+b)^2(x^2+y^2).$$

The maximum section will be made by a plane passing through the center of the ring and making an angle β such that $\sin\beta<[(a-b)/(a+b)]$.

Let $z=x\tan\beta$ be this plane.

$\therefore (x^2\sec^2\beta+y^2+ab)^2=(a+b)^2(x^2+y^2)$.

The area of this section is $(\pi/\sec^2\beta)(a^2-b^2)$.

This is a maximum when $\sec^2\beta$ is least, $\therefore \beta=0$.

$\therefore (x^2+y^2+ab)^2=(a+b)^2(x^2+y^2)$.

$\therefore$ the plane coincides with the xy plane and divides the ring into two equal parts. The section is included between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, area$=\pi(a^2-b^2)$.

MECHANICS.

64. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A cylindrical vessel, radius of vessel r and altitude h, is filled with water and rests on a horizontal plane. It is required to ascertain the maximum angle of elevation to which the plane may be raised without the vessel falling, allowing the coefficient of friction to be such as to prevent sliding, and the water to overflow as the plane is raised.

I. Solution by the PROPOSER.

Let MQ be the inclined plane; $ABCD$ a vertical section of the vessel; $ABFD$ a vertical section of the water when the vessel is on the point of turning over; $AB=2R$, the diameter of the vessel; $AD=H$, its altitude; and the angle $PMN=\theta$, maximum inclination of the plane. The quantity of water in the vessel at the time the vessel is on the point of turning over consists of the cylindrical part whose vertical section is $ABFE$ and the cylindrical ungula whose vertical section is EFD. Let V_2 be the volume of the cylindrical part, and V_1 the volume of the ungula. Let G_2 be the center of gravity of the cylindrical part, G_1 the center of gravity of the ungula, and G_0 the common center of gravity of both.